Introduction to New-Keynesian Business Cycle Modeling

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Introduction

- We analyze a model that is representative of the **new-Keynesian** class of business cycle models.
- The model is *Keynesian* because it assumes that nominal prices are not flexible; they adjust gradually in response to an exogenous shock to the economy.
- The model is *new* because it is built upon a solid microeconomic foundation.
- New-Keynesian models are useful for explaining the equilibrium relationships between inflation, output, and interest rates.

The Euler Equation

- Suppose that a *representative household* lives for two periods.
- The household receives utility from consuming goods in each period.
- The *lifetime utility* to the household from consuming C_0 in the first period and C_1 in the second is denoted by $U(C_0, C_1)$ and is written as:

$$U(C_0, C_1) = u(C_0) + \beta u(C_1), \tag{1}$$

where $u(\cdot)$ is the period utility function with $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $\beta \leqslant 1$.

Figure 1: **The household's flow of utility** in a single period as a function of consumption.

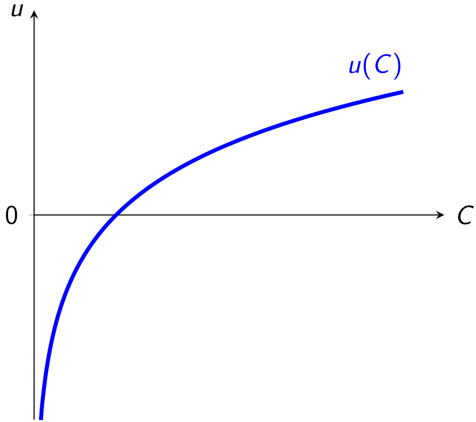
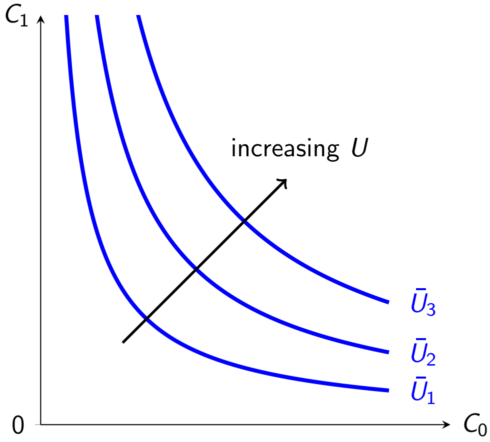


Figure 2: **The household's indifference curves** over combinations of consumption in periods 0 and 1.



The Euler Equation

- Household has no initial wealth
- Receives an income *endowment* of Y_0 in period 0 and Y_1 in period 1.
- In period 0, household chooses an amount of Y_0 to save S_1 given a real interest rate r_0.

The Euler Equation

- The household's budget constraints for periods 0 and 1 are:

$$C_0 = Y_0 - S_1, \tag{2}$$
$$C_1 = Y_1 + (1 + r_0)S_1. \tag{3}$$

- Combine the two period budget constraints to obtain the intertemporal budget constraint:

$$C_1 = Y_1 + (1 + r_0)Y_0 - (1 + r_0)C_0. \tag{4}$$

The Euler Equation

- The household's optimization problem is:

$$\max_{C_0, C_1, S_1} \; u(C_0) + \beta u(C_1) \tag{5}$$

$$\text{s.t.} \quad C_0 = Y_0 - S_1 \tag{6}$$

$$C_1 = Y_1 + (1 + r_0) S_1. \tag{7}$$

- Equivalent to:

$$\max_{S_1} \; u(Y_0 - S_1) + \beta u\left[Y_1 + (1 + r_0) S_1\right] \tag{8}$$

The Euler Equation

- Setting derivative of the objective function with with respect to S_1 equal to zero:

$$-u'(Y_0 - S_1) + (1 + r_0)\beta u'\left[Y_1 + (1 + r_0)S_1\right] = 0, \quad (9)$$

where $u'(\cdot)$ denotes the period marginal utility of consumption.

- Equation (11) is the *first-order condition* for the optimal choice of S_1.

- We can rewrite the first-order condition for S_1 in terms of consumption:

$$\boxed{u'(C_0) = \beta(1 + r_0)u'(C_1)} \qquad (10)$$

The Euler Equation

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$$-u'(Y_0 - S_1) + (1 + r_0)\beta u'\left[Y_1 + (1 + r_0)S_1\right] \quad = \quad 0, \tag{11}$$

  where $u'(\cdot)$ denotes the period marginal utility of consumption.

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- We can rewrite the first-order condition for S_1 in terms of consumption:

$$\boxed{u'(C_0) = \beta(1 + r_0)u'(C_1)} \tag{12}$$

The Euler Equation

- Given Y_0, Y_1, and r_0, the Euler equation (14) determines whether the household will borrow or save.

- If Y_0 is relatively large or if r_0 is relatively large, then the household will want to save.

Figure 3: **A Household that saves**. The period 0 endowment is large relative to the period 1 endowment

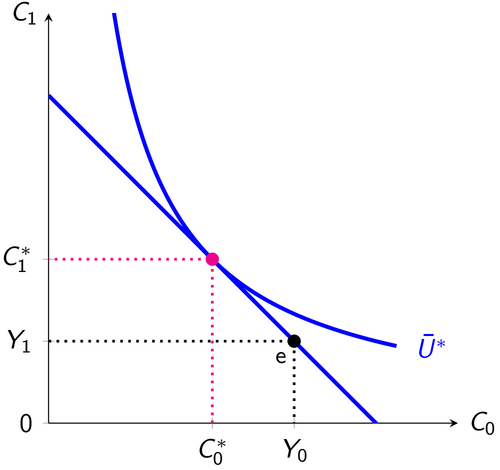
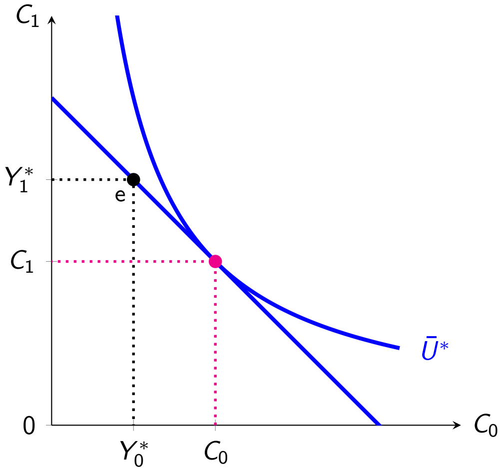


Figure 4: **A Household that borrows**. The period 0 endowment is small relative to the period 1 endowment



The Euler Equation

- We will assume that the period utility function is the natural logarithm of consumption:

$$u(C) = \log C. \tag{13}$$

- With this utility function, the household's Euler equation is expressed as:

$$\boxed{\frac{1}{C_0} = \frac{\beta(1+r_0)}{C_1}} \tag{14}$$

The Demand for Goods

- Assume no investment, government purchases, or net exports, so $Y = C$ in every period:

$$\frac{1}{Y_0} = \beta \frac{1}{Y_1}(1 + r_0) \tag{15}$$

- Next, take the log of each side to obtain a linear representation of the Euler equation:

$$\log Y_0 = \log \beta - \log Y_1 + \log(1 + r_0) \tag{16}$$

The Demand for Goods

- Rearranging and defining $\bar{r} \equiv -\log\beta$ as the natural rate of interest and $y \equiv \log Y$, obtain:

$$y_0 = y_1 - (r_0 - \bar{r}). \tag{17}$$

- Next:
  - The previous expression will hold for any two adjacent time periods t and $t+1$
  - Incorporate uncertainty: replace y_1 with its expectation conditional on period 0 information $E_0 y_1$
  - Append an exogenous demand shock
- Finally:

$$\boxed{y_t = E_t y_{t+1} - (r_t - \bar{r}) + g_t} \tag{18}$$

Equilibrium

- The complete set of equilibrium conditions:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - (r_{t+1} - \bar{r}) + g_t && (19) \\
i_t &= r_t + E_t \pi_{t+1} && (20) \\
i_t &= \bar{r} + \pi^T + \phi_\pi \left(\pi_t - \pi^T\right) + \phi_y \left(y_t - \bar{y}\right) + v_t && (21) \\
\pi_t - \pi^T &= \beta \left(E_t \pi_{t+1} - \pi^T\right) + \kappa(y_t - \bar{y}) + u_t, && (22)
\end{aligned}
$$

- The equations are: dynamic IS equation, Fisher equation, monetary policy rule, new-Keynesian Phillips curve or dynamic AS equation.

References