Technical Appendix: Risk Averse Banks and Endogenous Fluctuations in Excess Reserves

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1 Complete Set of Equilibrium Conditions

$$Y_t = Z_t K_t^{\alpha} H_t^{(1-\alpha)\Omega} \tag{1}$$

$$Y_t^f = \frac{1}{S_t} Y_t \tag{2}$$

$$K_{t+1} = \Psi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta) K_t \tag{3}$$

$$Q_t = \left[\Psi'\left(\frac{I_t}{K_t}\right)\right]^{-1} \tag{4}$$

$$R_t^K = E_t \frac{\frac{1}{X_t}\frac{\alpha Y_t}{K_t} + \bar{Q}_t(1-\delta)}{Q_{t-1}} \tag{5}$$

$$Q_t \Psi\left(\frac{I_t}{K_t}\right) - \frac{I_t}{K_t} - \left(\bar{Q}_t - Q_t\right) = 0 \tag{6}$$

$$(1-\alpha)\Omega \frac{Y_t}{H_t} = X_t W_t \tag{7}$$

$$N_{t+1} = \gamma \left[1 - \Gamma_t\right] R_t^K Q_{t-1} K_t + (1-\alpha)(1-\Omega) Z_t K_t^{\alpha} H_t^{(1-\alpha)\Omega} / X_t \tag{8}$$

$$\lambda_t = C_t^{-1} \tag{9}$$

$$C_t^e = (1-\gamma)\left[1-\Gamma_t\right] R_t^K Q_{t-1} K_t \tag{10}$$

$$Y_t^f = C_t + C_t^e + I_t + G_t + \mu \Upsilon_t R_t^K Q_{t-1} K_t \tag{11}$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1}\right\} R_t \tag{12}$$

$$W_t C_t^{-1} = \zeta (1 - H_t)^{-1} \tag{13}$$

$$R_t^n = R_t E_t \Pi_{t+1} \tag{14}$$

$$\frac{R_t^n}{\bar{R}^n} = \left(\frac{R_{t-1}^n}{\bar{R}^n}\right)^{\rho_r} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t^f}{\bar{Y}^f}\right)^{\phi_y} e^{v_t} \tag{15}$$

$$Q_t K_{t+1} - N_{t+1} + \frac{M_t^{ex}}{P_t} = (1-\rho)\zeta_{\text{D}} C_t \left(\frac{R_t^n}{R_t^n - R_t^D}\right) \tag{16}$$

$$\frac{1 - R_t^D}{1-\rho} E_t \left\{\frac{1}{1+\Pi_{t+1}} \tilde{u}'\left(\Phi_{t+1}\right)\right\} + \frac{\mu^{ex}}{M_t^{ex}/P_t} = 0 \tag{17}$$

$$\frac{B_t}{P_t} = Q_t K_{t+1} - N_{t+1} \tag{18}$$

And:

$$1 = \theta \Pi_t^{-1+\epsilon} + (1-\theta)\tilde{P}_t^{1-\epsilon} \tag{19}$$

$$x_t^1 = \tilde{P}_t^{-1-\epsilon} \frac{Y_t^f}{X_t} + \theta \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{\epsilon} \left(\frac{\tilde{P}_t}{\tilde{P}_{t+1}} \right)^{-1-\epsilon} x_{t+1}^1 \right\} \tag{20}$$

$$x_t^2 = \tilde{P}_t^{-\epsilon} Y_t^f + \theta \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{\epsilon-1} \left(\frac{\tilde{P}_t}{\tilde{P}_{t+1}} \right)^{-\epsilon} x_{t+1}^2 \right\} \tag{21}$$

$$\frac{\epsilon}{\epsilon - 1} x_t^1 = x_t^2 \tag{22}$$

$$S_t = (1-\theta)\tilde{P}_t^{-\epsilon} + \theta \Pi_t^{\epsilon} S_{t-1} \tag{23}$$

And:

$$\Gamma_t = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\tilde{Z} - \sigma_{\omega,t}}{\sqrt{2}}\right)\right] + \bar{\omega}\frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{\tilde{Z}}{\sqrt{2}}\right)\right] \tag{24}$$

$$\Gamma_t' = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{\tilde{Z}}{\sqrt{2}}\right)\right] \tag{25}$$

$$\Upsilon_t = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\tilde{Z} - \sigma_{\omega,t}}{\sqrt{2}}\right)\right] \tag{26}$$

$$\Upsilon_t' = \frac{1}{\sigma_{\omega,t}\sqrt{2\Pi}} \exp\left\{ -\frac{\tilde{Z}^2}{2} \right\} \tag{27}$$

$$\tilde{Z}_t = \frac{\log \bar{\omega}_t + \sigma_{\omega,t}^2/2}{\sigma_{\omega,t}} \tag{28}$$

$$\begin{aligned} & E_t \left\{ \left(1 - \Gamma_{t+1}\right) R_{t+1}^K - \frac{\Gamma_{t+1}'}{1+\Pi_{t+1}} \frac{\bar{R}_t N_{t+1}}{Q_t K_{t+1}} \right\} \\ & \quad + \tilde{\lambda}_t \cdot E_t \left\{ \tilde{u}'\left(\Phi_{t+1}\right) \left[\left(\Gamma_{t+1}' - \mu \Upsilon_{t+1}'\right) \frac{\bar{R}_t N_{t+1}}{(1+\Pi_{t+1}) Q_t K_{t+1}} \right. \right. \\ & \qquad \left. \left. + \ \left(\Gamma_{t+1} - \mu \Upsilon_{t+1}\right) R_{t+1}^K - \frac{R_t^D - \rho}{(1+\Pi_{t+1})(1-\rho)} \right] \right\} = 0 \end{aligned} \tag{29}$$

$$E_t \left\{ \Gamma_{t+1}' \frac{1}{1+\Pi_{t+1}} \right\} - \tilde{\lambda}_t \cdot E_t \left\{ \tilde{u}'\left(\Phi_{t+1}\right) \left(\Gamma_{t+1}' - \mu \Upsilon_{t+1}'\right) \frac{1}{1+\Pi_{t+1}} \right\} = 0 \tag{30}$$

$$E_t \left\{ \tilde{u}\left(\Phi_{t+1}\right) \right\} - E_t \left\{ \tilde{u}\left(\frac{1}{1+\Pi_{t+1}} \frac{1 - R_t^D}{1-\rho} \frac{M_t^{ex}}{P_t} \right) \right\} = 0 \tag{31}$$

$$\bar{\omega}_t (1+\Pi_t) R_t^K Q_{t-1} K_t = \bar{R}_t (Q_{t-1} K_t - N_t) \tag{32}$$

And:

$$\Phi_{t+1} = \frac{1}{1+\Pi_{t+1}}\left[\chi_{t+1}\left(\frac{B_t}{P_t}\right) - \frac{R_t^D-\rho}{1-\rho}\frac{B_t}{P_t} + \frac{1-R_t^D}{1-\rho}\frac{M_t^{ex}}{P_t}\right] \tag{33}$$

$$\chi_{t+1}\left(\frac{B_t}{P_t}\right) = \left(\Gamma_{t+1} - \mu\Upsilon_{t+1}\right)R_{t+1}^K(1+\Pi_{t+1})Q_tK_{t+1} \tag{34}$$

Where:

$$\tilde{u}\left(\Phi\right) = -\exp\left\{-\xi_\Phi\Phi\right\} \tag{35}$$

$$\tilde{u}'\left(\Phi\right) = \xi_\Phi\exp\left\{-\xi_\Phi\Phi\right\} \tag{36}$$

And the exogenous processes:

$$\log Z_t = \rho_Z\log Z_{t-1} + \varepsilon_t^Z \tag{37}$$

$$\log G_t = (1-\rho_g)\log\bar{G} + \rho_G\log G_{t-1} + \varepsilon_t^G \tag{38}$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{39}$$

$$\tag{40}$$

And the stochastic volatility specification for $\sigma_{\omega,t}$:

$$\sigma_{\omega,t} = \left(1-\rho_\sigma\right)\bar{\sigma}_\omega + \rho_\sigma\sigma_{\omega,t-1} + \exp\left(\sigma_{\sigma,t}\right)\varepsilon_t^\sigma \tag{41}$$

$$\sigma_{\sigma,t} = \left(1-\rho_{\sigma\sigma}\right)\bar{\sigma}_\sigma + \rho_{\sigma\sigma}\sigma_{\sigma,t-1} + \varepsilon_t^{\sigma\sigma} \tag{42}$$

And the investment adjustment function:

$$\Psi\left(\frac{I_t}{K_t}\right) = \frac{1}{1-\psi}\left(\frac{I_t}{K_t}\right)^{1-\psi}\left(\frac{I}{K}\right)^{\psi} - \frac{\psi}{1-\psi}\left(\frac{I}{K}\right) \tag{43}$$

2 Calibration and Steady State Computation

Take as given values for α, Ω, θ, ϵ, ψ, $F(\bar{\omega}|\sigma_\omega^2)$, ρ, and ξ_Φ. Also, suppose that the following average first moments are known:

- average Π
- average H
- average R^n
- average R^D (deposit rate)
- average I/Y^f (investment to GDP ratio)
- average G/Y^f (government consumption to GDP ratio)
- average $(K-N)/N$ (debt to equity ratio)
- average M^{ex}/D (excess reserves to deposits ratio)

Compute the steady state and calibrate the values of parameters implied by the empirical values collected above. The process goes like this.

$$\Pi = \text{average } \Pi \tag{44}$$
$$H = \text{average } H \tag{45}$$
$$R^n = \text{average } R^n \tag{46}$$
$$R^D = \text{average } R^D \text{ (deposit rate)} \tag{47}$$
$$R = \frac{R^n}{\Pi} \tag{48}$$

Calibrate β:

$$\beta = \frac{1}{R} \tag{49}$$

Compute some steady state values directly.

$$Q = 1 \tag{50}$$
$$\bar{Q} = Q \tag{51}$$
$$\tilde{P} = \left(\frac{1 - \theta\Pi^{-1+\epsilon}}{1-\theta}\right)^{1/(1-\epsilon)} \tag{52}$$
$$S = \frac{(1-\theta)\tilde{P}^{-\epsilon}}{1-\theta\Pi^{\epsilon}} \tag{53}$$
$$X = \left(\frac{1-\theta\beta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon}}\right)\left(\frac{\epsilon}{\epsilon-1}\right)\tilde{P}^{-1} \tag{54}$$
$$R^K = \frac{\alpha\delta S}{X \cdot (\text{average investment to GDP ratio})} + 1 - \delta \tag{55}$$
$$K = \left(\frac{\alpha H^{(1-\alpha)\Omega}}{X(R^K + \delta - 1)}\right)^{1/(1-\alpha)} \tag{56}$$
$$I = \delta K \tag{57}$$
$$Y = K^{\alpha} H^{(1-\alpha)\Omega} \tag{58}$$
$$Y^f = \frac{Y}{S} \tag{59}$$

And:

$$x_1 = \frac{\tilde{P}^{-1-\epsilon} Y^f}{X(1-\theta\beta\Pi^\epsilon)} \tag{60}$$

$$x_2 = \frac{\epsilon}{\epsilon - 1} x_1 \tag{61}$$

$$W = \frac{(1-\alpha)\Omega Y}{XH} \tag{62}$$

$$N = \frac{K}{1 + \text{average debt to equity ratio}} \tag{63}$$

$$\frac{B}{P} = QK - N \tag{64}$$

Calibrate steady state government purchases:

$$\bar{G} = Y^f \cdot (\text{average government purchases to GDP ratio}) \tag{65}$$

Solve the following system numerically for Γ, Γ', Υ, Υ', $\tilde{Z}$, σ_ω, $\bar{\omega}$, $\bar{R}$, and μ:

$$\Gamma = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\tilde{Z} - \sigma_\omega}{\sqrt{2}}\right)\right] + \bar{\omega}\frac{1}{2}\left[1 - \text{erf}\left(\frac{\tilde{Z}}{\sqrt{2}}\right)\right] \tag{66}$$

$$\Gamma' = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\tilde{Z}}{\sqrt{2}}\right)\right] \tag{67}$$

$$\Upsilon = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\tilde{Z} - \sigma_\omega}{\sqrt{2}}\right)\right] \tag{68}$$

$$\Upsilon' = \frac{1}{\sigma_\omega\sqrt{2\Pi}} \exp\left\{-\frac{\tilde{Z}^2}{2}\right\} \tag{69}$$

$$\tilde{Z} = \frac{\log\bar{\omega} + \sigma_\omega^2/2}{\sigma_\omega} \tag{70}$$

$$\bar{\omega} = \frac{\bar{R}(QK - N)}{R^K QK\Pi} \tag{71}$$

$$0 = (1-\Gamma)\,R^K - \frac{\Gamma'}{\Pi}\frac{\bar{R}N}{QK} + \frac{\Gamma'}{\Gamma' - \mu\Upsilon'}\left[(\Gamma' - \mu\Upsilon')\frac{\bar{R}N}{\Pi QK} + (\Gamma - \mu\Upsilon)R^K \right. \tag{72}$$

$$\left. - \frac{(\Gamma - \mu\Upsilon)R^K(1+\pi)QK}{QK - N}\right] \tag{73}$$

$$\frac{R^D - \rho}{1 - \rho} = \left(\frac{(1-\Gamma)(\Gamma' - \mu\Upsilon')}{\Gamma'} + \Gamma - \mu\Upsilon\right) R^K\Pi \tag{74}$$

$$F(\bar{\omega}|\sigma_\omega^2) = \text{average default rate} \tag{75}$$

Then calibrate γ:

$$\gamma = \frac{N - (1-\alpha)(1-\Omega)K^{\alpha}H^{(1-\alpha)\Omega}/X}{[1-\Gamma]R^{K}QK} \tag{76}$$

Compute more steady state quantities directly:

$$C^{e} = (1-\gamma)(1-\Gamma)R^{K}QK \tag{77}$$

$$C = Y^{f} - C^{e} - I - G - \mu\Upsilon R^{K}QK \tag{78}$$

$$\lambda = C^{-1} \tag{79}$$

$$\chi = \left(\frac{R^{d}-\rho}{1-\rho}\right)\frac{B}{P} \tag{80}$$

Compute steady state excess reserves based on observed historic ratio of excess reserves to deposits:

$$\frac{M^{ex}}{P} = \frac{\text{average excess reserve to deposit ratio}}{D} \tag{81}$$

Compute more steady state quantities directly:

$$\Phi = \left(\chi - \frac{R^{d}-\rho}{1-\rho}\frac{B}{P} + \frac{1-R^{d}}{1-\rho}\frac{M^{ex}}{P}\right)/\Pi \tag{82}$$

$$\tilde{u}\left(\Phi\right) = -\exp(-\xi_{\Phi}\Phi) \tag{83}$$

$$\tilde{u}'\left(\Phi\right) = \xi_{\Phi}\exp(-\xi_{\Phi}\Phi) \tag{84}$$

$$\tilde{\lambda} = \frac{\Gamma'}{(\Gamma' - \mu\Upsilon')\tilde{u}'\left(\Phi\right)} \tag{85}$$

Calibrate penalty function parameter

$$\mu_{x} = \frac{R^{d}-1}{1-\rho}\frac{\tilde{u}'\left(\Phi\right)}{\Pi}\frac{M^{ex}}{P} \tag{86}$$

Calibrate preference parameter on deposits

$$\zeta_{d} = \left(\frac{QK - N + M^{ex}/P}{1-\rho}\right)C\left(\frac{R^{n}-R^{d}}{R^{n}}\right) \tag{87}$$

Calibrate preference parameter on leisure

$$\zeta_{h} = WC^{-1}(1-H) \tag{88}$$