Online Appendix for "Revisiting Unemployment in Intermediate Macro: A New Approach for Teaching Diamond-Mortensen-Pissarides"

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This online appendix contains four sections. Section 1 discusses more details about the matching function. Section 2 connects the DMP model and Solow Growth model. Section 3 discusses the dynamics in the DMP model. Finally, Section 4 provides more detailed instructions on how to use the DMP model simulation tool.[1]

1 Additional details about the matching function

Throughout the paper, we assume that the aggregate matching function H is Cobb-Douglas but other specifications are permissible provided:

1. The matching function is increasing in each of its arguments, i.e.: $\partial H/\partial U > 0$ and $\partial H/\partial V > 0$. We also assume that $H(0, V) = H(U, 0) = 0$, i.e., there cannot be any matches if there are no unemployed workers or if there are no vacant jobs.

2. The matching function exhibits constant returns to scale. The implication of CRS is that the size of the economy is not relevant for the matching process.

3. The number of hires cannot exceed the number of vacancies or the number of unemployed workers, i.e.: $H(U, V) \leq U$ and $H(U, V) \leq V$.

Instructors can give examples of other functional forms that are used in the literature:

$$H(U, V) = \frac{AUV}{U + V}$$
$$H(U, V) = AV(1 - e^{\frac{U}{V}})$$
$$H(U, V) = A \min \{U, V\}.$$

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[1]The simulation tool is available online at http://www.briancjenkins.com/dmp-model/simulation-tool.html.

The second functional form is called the urn-ball matching function. The name has its roots in the urn-ball game where there are U number of balls and V number of urns. Balls are placed at random in the urns. If U and V are large, then it can be shown mathematically that the number of urns with at least one ball is given by the expression provided above. To understand how it relates to the labor market, one can think of the balls as job applications and the receiving urns as vacancies with a single job. The third example is the Leontief matching function, where all agents on the short side of the market are matched. After introducing the alternative matching functions, it can be useful to assign as an exercise to the students, to check whether the above matching functions satisfy properties (1)-(3) above.

2 DMP meets Solow

In notes throughout the text, we made several analogies between the DMP model and the Solow growth Model. In this section we combine the two models to study how changes in fundamentals such as the savings rate and the depreciation rate of capital affect unemployment. We reconcile these two models by assuming that the employed workers in the DMP model serve as the labor input which is combined with capital to produce output in the Solow model. Thus, the workers' productivity, y can simply be thought of as the output per employed worker in the Solow model. We assume that the unemployment benefit in the DMP model b is an exogenously given stream of income that doesn't come from any production within the economy.[2]

Suppose that the size of the labor force is constant and is denoted by L. There is an aggregate production function which as two inputs, capital at time t, K_t, and employed workers at time t, $E_t = (1 - u_t)L$. Output, Y_t, is given by a standard neoclassical production function:

$$Y_t = F(K_t, E_t). \tag{1}$$

Assume that the depreciation rate of capital is constant across time and is given by $0 < \delta < 1$. The law of motion for capital is given by:

$$K_{t+1} - K_t = sF(K_t, E_t) - \delta K_t, \tag{2}$$

where s is the savings rate in the Solow model. We define the steady state in this model as the steady state in the Solow economy as well as in the DMP model. Thus, in a steady state, $K_{t+1} = K_t = K$ and $u_{t+1} = u_t = u$ which also implies $E_t = E_{t+1} = E$. From (2), in a steady state:

$$sF(K, E) = \delta K. \tag{3}$$

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[2]One could also assume that b is financed through lump-sum taxes on output. In either case, this does not distort capital accumulation decisions. We opt for the option where b is an exogenous stream of income to keep the exposition as simple as possible while still illustrating a connection between the DMP and Solow models.

Define the capital per employed worker $k \equiv K/E$. Dividing (3) by E and imposing that the production function F has constant returns to scale, we obtain:

$$sf(k) = \delta k, \tag{4}$$

where $f(\cdot)$ is the production function in intensive form. Equation (4) is the familiar condition from the Solow growth model which gives the steady state capital per worker. By making the connection to the DMP model, we see that the steady state aggregate capital stock, $K = k(1 - u)L$ will be a function of the unemployment rate, u, as well as k determined through (4). Thus, for a given level of unemployment, the solution to (4) will give the steady state aggregate capital stock K.

Now that we have a condition to determine the aggregate capital stock K, we solve for the steady state unemployment rate. Recall from the DMP model:

$$u = \frac{\lambda^2 \kappa}{\lambda^2 \kappa + A^2(1 - \beta)(y - b)}. \tag{5}$$

Denote output per employed worker $f(k)$ by y. For a simple example, we assume that following form of the intensive production function $f(k) = Zk^{1/3}$, where Z is the total factor productivity parameter. From (4):

$$k = \left(\frac{sZ}{\delta}\right)^{3/2}, \tag{6}$$

$$y = \left(\frac{sZ^3}{\delta}\right)^{1/2}. \tag{7}$$

We can see that the steady state capital stock per employed worker is increasing in the savings rate and decreasing in the depreciation rate. Substituting (7) into (5) gives:

$$u = \frac{\lambda^2 \kappa}{\lambda^2 \kappa + A^2(1 - \beta)\left(\sqrt{\frac{sZ^3}{\delta}} - b\right)}. \tag{8}$$

We see that the unemployment rate u is decreasing in the savings rate s and increasing in the depreciation rate of capital δ. This is because an increasing in the savings rate s corresponds to higher per-capita capital and per-capita output. On the other hand, a decrease in the depreciation rate of capital δ leads to increase in per-capital capital and per-capital output y in the steady state. In the DMP model, it has been established earlier that the unemployment rate u is decreasing in worker productivity y. Therefore, unemployment is decreasing in the savings rate and is increasing in the depreciation rate.

3 Dynamics in the DMP model

The main text focused on the steady state and how changes in exogenous factors affect the steady state equilibrium. But we can also use the DMP model to study how the labor market transitions between steady states. While it's not necessary to cover the dynamics of the DMP model, doing so can give students a better intuition for the DMP model and dynamic economic models more generally.

Our objective is to characterize the dynamics for the three endogenous variables: market tightness, the real wage, and the unemployment rate. To begin, we note that market tightness and the real wage are determined by the vacancy supply condition:

$$\theta_t = \left[\frac{A}{\kappa} \left(\frac{y - w_t}{\lambda} \right) \right]^2, \tag{9}$$

and the wage equation:

$$w_t = \beta(y + \kappa\theta_t) + (1 - \beta)b. \tag{10}$$

Eliminating w_t by substituting (10) into (9) gives

$$\theta_t = \left[\frac{A}{\kappa} \left(\frac{(1-\beta)(y-b) - \beta\kappa\theta_t}{\lambda} \right) \right]^2, \tag{11}$$

From equation (11) it is apparent that market tightness is always equal to its steady state value which solves (11). That is, market tightness jumps immediately to its new steady state value in response to a change in one of the exogenous variables of the model. Moreover, the wage w_t and the job finding rate $f_t = A\sqrt{\theta_t}$ also jump to new steady state values.

Next, we characterize unemployment dynamics. Recall the law of motion

$$u_{t+1} = (1 - A\sqrt{\theta_t} - \lambda)u_t + \lambda, \tag{12}$$

where we have substituted $f_t = A\sqrt{\theta_t}$. Together, equations (10), (11), and (12) characterize the dynamic behavior of the unemployment rate, market tightness, and the real wage.

Equation (12) implies that the unemployment rate will transition monotonically toward its steady state value as long as $0 \leq 1 - A\sqrt{\theta_t} - \lambda \leq 1$ or equivalently, if $0 < f_t + \lambda \leq 1$. Using monthly data from the BLS from January 1, 1948 to September 1, 2016, the average unemployment rate in the US was about 5.81% and an average unemployment duration of 3.94 months implies an average monthly job finding rate of about 25.4% over the same period. In the steady state: $u = \frac{\lambda}{f+\lambda}$, implying that the monthly separation rate from the US from January 1948 to September 2016 has been about 1.57%. Therefore, the long run average value for $f + \lambda$ has been about 0.2697. Since this values is below 1, we know that unemployment rate transitions monotonically.

3.1 An increase in productivity

We consider the dynamic consequences of an increase in productivity y. Specifically, we assume that the labor market is in a steady state starting at period $t = 0$ and that in period $t = 2$, productivity rises permanently from y to a new value y'. The increase in productivity will increase the real wage because the marginal product of labor rises. Steady state market tightness will also rise as firms create additional vacancies on account of the rising expected profit from opening a vacancy. Unemployment will fall because the rising market tightness drives up the job finding rate.
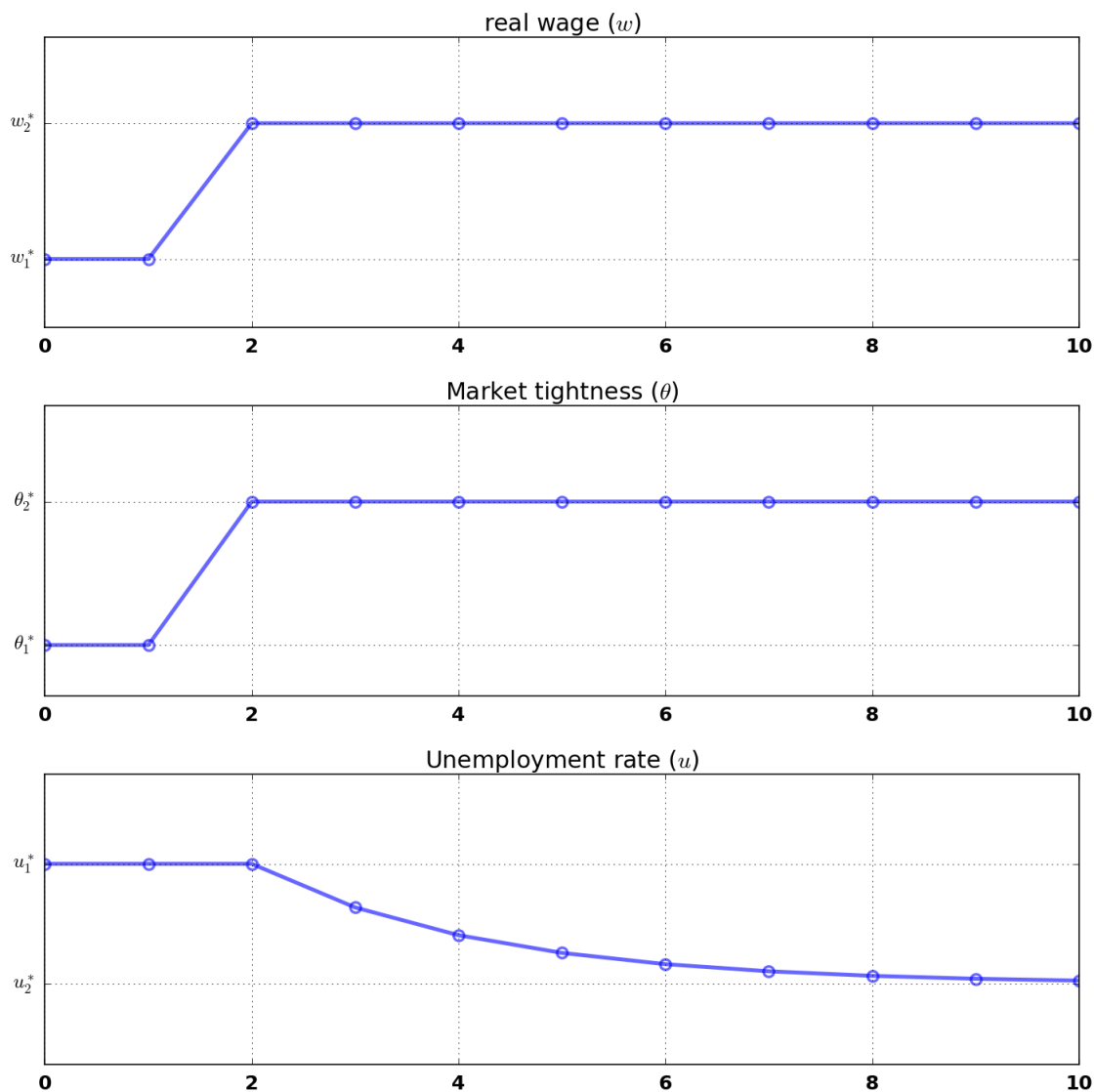

Figure 1: Dynamic responses of the real wage, market tightness, and unemployment following a permanent increase in productivity y.

Figure 1 depicts the transition paths of market tightness, the real wage, and the unem-

ployment rate to their respective new steady states. At the date that productivity rises, market tightness and the real wage both jump immediately to their new steady state values. The unemployment rate, however, transitions gradually and monotonically to its new (lower) steady state value starting in the period *after* the increase in productivity.

4 Instructions for the DMP simulation tool

4.1 Overview

The DMP Model Simulation Tool is a web-based resource for visualizing changes in the DMP model equilibrium. The tool is available at:

$$\text{http://www.briancjenkins.com/dmp-model/simulation-tool.html.}$$

The simulation tool allows the user to pick an exogenous variable from the model, specify whether the value of that variable is to increase or decrease, and then see how the relevant curves of the model shift. The user may optionally choose to have the transition of the unemployment rate rendered dynamically on the Beveridge curve graph. The tool is an ideal resource from homework assignments because the generated figures can be exported to one of several standard image formats to be included in a document.

4.2 Instructions

1. Configure your simulation.

    - Choose the exogenous variable to change: Select the radio button associated with the variable that you wish to change. *Note that you may only choose one.*
    - Direction of change: Select the radio button indicating whether you would like to see the selected variable increase or decrease.
    - Show unemployment transition: Indicate whether you would like to see the dynamic path of the unemployment rate plotted on the Beveridge curve figure.

2. Click "Submit" to see the computed simulation.

3. *Optional.* Download the simulated images by clicking the icon in the top-right of each image and selecting the desired image format.

4. Click "Reset" to reset the figures and configurations.

4.3 Example

Figure 2 contains an example illustrating how to use the tool. We illustrate an increase in productivity. Increasing productivity shifts the WS and VS curves to the right, raises the wage and market tightness, and reduces unemployment. Note that in this example, we opted to not show the unemployment transition.
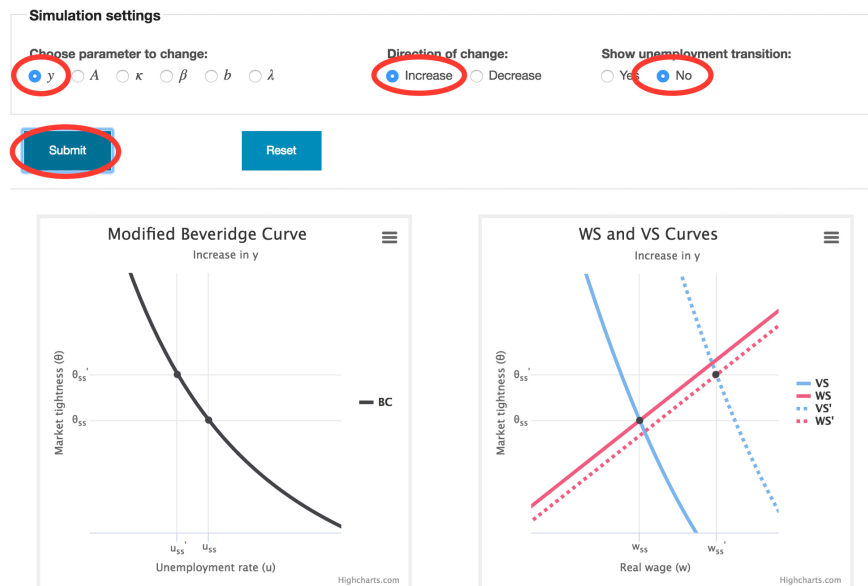

Figure 2: An example of how to use the DMP model simulation tool.